$\bar{p}$ Note # 485

BUNCHED BEAM COOLING STUDIES IN THE ACCUMULATOR

Sept. 4-8, 1989

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INTRODUCTION

Bunched beam cooling studies were performed on the Accumulator during Sept. 4-8, 1989. A proton beam from the BOOSTER was injected into the core and was bunched with the h=2 or h=84 RF systems. Problems with the h=84 RF system prevented detailed studies of bunch beam cooling at this harmonic. The rest of this report will deal only with a beam bunched at the h=2 harmonic. Because the vertical plane has a lower chromaticity than the horizontal plane, only vertical betatron cooling was studied in detail. However, results on cooling in the horizontal plane are qualitatively similar.

BEAM PREPARATION

As stated earlier, a proton beam of approximately 10 mA was injected from the BOOSTER into the Accumulator injection orbit. The beam was then moved to the central orbit with the h=2 RF systems and subsequently scraped to a momentum spread that could be accommodated by the 2-4 GHz momentum cooling system. The final beam intensity was about 5 mA. The beam was unbunched and the momentum spread was reduced further to approximately 0.06% by the 2-4 GHz central orbit momentum cooling system. Also the horizontal emittance was reduced by the 4-8 GHz core horizontal betatron system.

DC BEAM COOLING MEASUREMENTS

With the beam in a DC or coasting state, the following data was taken:

1. The frequency spectrum of the 126th longitudinal Schottky band.
2. The DC beam current.
3. The open loop and closed loop amplitudes of the vertical betatron Schottky bands at 4.0 GHz.

4. The vertical emittance as a function of time with the 4-8 GHz vertical cooling system on.

From these measurements, the mixing factor, cooling gain, signal/noise, and cooling bandwidth could be determined.

DC BEAM CALCULATIONS

Mixing Factor

The mixing factor is calculated from the longitudinal Schottky spectrum by assuming that the Schottky line is a guassian function of frequency.

$$\psi(f) = \frac{N}{\sqrt{2\pi}\,\sigma_f} e^{-\frac{f^2}{2\sigma_f^2}}$$

The mixing factor is defined as:

$$M(f) = \frac{\psi(f)_{peak}}{\psi_{average}}$$

Because the 10 dB full width of the guassian is:

$$\Delta f_{10\,dB} = 4.292\,\sigma_f$$

and using the fact that:

$$\sigma_f(f) = \frac{f}{f_o}\sigma_f(f_o)$$

the average mixing factor for a cooling system that operates over a frequency range from 4 GHz to 4+W GHz is given as:

$$M_{average} = \frac{85.31 \times 10^3}{\Delta f(hz)_{10\,dB_{h=126}}\left(4 + \frac{W(GHz)}{2}\right)}$$

Noise to Signal ratio

The average noise/signal ratio of a betatron Schottky band is found from measuring the peak Signal+Noise of a betatron Schottky line and the average noise floor. The peak Signal/Noise is:

$$\frac{S_{peak}}{N_{avg}} = \frac{(S+N)_{peak}}{N_{avg}} - 1$$

The average noise to signal in a Schottky band is:

$$U = \frac{1}{2}\frac{N_{avg}}{S_{avg}} = \frac{1}{2}\frac{N_{avg}}{S_{peak}}\frac{S_{peak}}{S_{avg}}$$

The factor of 1/2 appears because there are two betatron Schottky lines per band. The ratio between the peak signal and the average signal is the mixing factor. Thus the average noise to signal in a Schottky band is:

$$U = \frac{1}{2}\frac{N_{avg}}{(S+N)_{peak} - N_{avg}}M(f)$$

Since all the signal to noise measurements were done at 4 GHz, the average noise to signal is:

$$U = \frac{1}{2}\frac{N_{avg}}{\left((S+N)_{peak} - N_{avg}\right)}\frac{10.66\mathrm{x}10^{3}}{\Delta f(\mathrm{Hz})_{10\mathrm{dB}_{h=126}}}$$

Cooling Gain

The signal suppression is the ratio between the open loop gain and the closed loop gain. This ratio can be written as:

$$SS(\mathrm{dB}) = 20\log_{10}\left(1+\frac{g}{g_{opt}}\right)$$

The cooling gain is found by inverting the above equation:

$$\frac{g}{g_{opt}} = 10^{\frac{SS(dB)}{20}} - 1$$

Cooling Bandwidth

The cooling time, τ, is given as:

$$\frac{1}{\tau} = \frac{W}{N_p}\left(2g - g^2(M+U)\right)$$

where N_p is the number of protons in the machine. The number of particles in the Accumulator is equal to:

$$N_p = 1x10^{10}\, I_{beam}(mA)$$

The cooling time is a minimum for a cooling gain given by:

$$g_{opt} = \frac{1}{M+U}$$

The equation for the cooling time can then be rewritten as:

$$\frac{1}{\tau} = \frac{W}{N_p(M+U)}\left(\frac{g}{g_{opt}}\right)\left(2 - \frac{g}{g_{opt}}\right)$$

Every quantity in this equation has been measured except for the cooling bandwidth. This equation can be inverted to determine W. Since the mixing factor is a function of cooling bandwidth, the resulting equation will be quadratic in W.

$$aW^2 + bW + c = 0$$

where W has units of GHz and:

$$a = \frac{\tau(Sec)}{10 I_{beam}(mA)}\, \Delta f(Hz)_{10db_{h=126}}\left(\frac{g}{g_{opt}}\right)\left(2 - \frac{g}{g_{opt}}\right)$$

$$b = 8a - U\,\Delta f\,(Hz)_{10dB_{h=126}}$$

$$c = -170.62 \times 10^{3} - 8U\,\Delta f(Hz)_{10dB_{h=126}}$$

Coasting Beam Calculations

The results of the measurements and the above calculations are summarized in the following table:

10 dB full width of Longitudinal Schottky band at h=126 (Hz)	805
Revolution Frequency (Hz)	628857
Signal Suppression (dB)	3.59
g/gopt	0.51
Schottky Line Noise to Signal at 4 GHz	0.16
Beam Current (mA)	5.81
Cooling time (Sec)	627
Cooling BandWidth (GHz)	2.46
Mixing Factor at Center Frequency	20.25

The cooling bandwidth is not 4 GHz because the gain equalizer for the coaxial trunk line had not been installed as of September 1989. Thus, the Schottky signal at the higher frequencies of 4-8 GHz band was severely attenuated.

Bunched beam measurements

The h=2 RF system (ARF3) was "adiabatically" raised to several different RF voltages. At each of these RF voltages, the vertical system was placed 180° from its optimum phasing and the beam was heated to a fairly large vertical emittance. The heated vertical emittance was approximately the same for each of the RF voltages. After the beam was heated, the vertical cooling system was returned to its proper phasing.

At each RF voltage, the vertical emittance as a function of time was measured with the 4-8 GHz vertical cooling system on. Along with each vertical emittance measurement the following data was also taken:

1. The ARF3 fanback voltage.
2. The bunch time domain structure was recorded using the resistive wall monitor. (This measurement was taken immediately before and after the vertical emittance measurement.)
3. The spectrum of the longitudinal Schottky band at h=126. (This measurement was taken immediately before and after the vertical emittance measurement.)
4. The open loop and closed loop amplitudes of the vertical betatron Schottky bands at 4.0 GHz.

These measurements were repeated for two different longitudinal emittances; a "hot" and a "cool" longitudinal emittance. The "cool" longitudinal emittance was obtained by engaging the 2-4 GHz momentum cooling system with an attenuation setting of 13dB for 10 minutes after the "hot" longitudinal emittance data was taken.

Bunched Beam Analysis

Because the Schottky line at h=126 has a large coherent signal with a bunched beam, the mixing factor derived from this information might be inaccurate. However, the mixing factor can be determined from analyzing the time domain response of the bunched beam.

The RMS half width, σ_T, can be determined by measuring the 1/2 maximum half width of a bunch:

$$\sigma_T = \Delta t_{1/2} \sqrt{\frac{-1}{2\ln\left(\frac{1}{2}\right)}}$$

The RMS half width in RF phase is equal to:

$$\sigma_\phi = 2\pi\sigma_T h f_o$$

where h is the RF harmonic number (h=2 for our case) and f_0 is the beam revolution frequency. The RMS half width of the momentum spread can be found by considering the orbit of a single particle in phase space with a maximum phase error of σ_ϕ. The orbit is governed by:

$$\Omega^2\left(\cos(\phi) - \cos\left(\sigma_\phi\right)\right) = \frac{\left(\omega_{RF}\eta\right)^2}{2}\left(\frac{\Delta p}{p}\right)^2$$

where Ω is the radial synchrotron frequency:

$$\Omega^2 = \frac{\omega_{RF}^2 \eta e V_{RF} \cos\left(\psi_s\right)}{\beta c p 2\pi h}$$

ψ_s is equal to -180^o for a stationary bucket. The momentum for the central orbit is 8.837 GeV/c; β = 0.994; and η = -0.022. The maximum value of the momentum error occurs when $\phi = 0$:

$$\left(\frac{\Delta p}{p}\right)_{RMS} = \frac{2\Omega}{\omega_{RF}\eta} \sin\left(\frac{\sigma_\phi}{2}\right)$$

The RMS frequency spread at the first Schottky band (h=1) is equal to:

$$\left(\frac{\Delta f_o}{f_o}\right)_{RMS} = \frac{2\Omega}{\omega_{RF}} \sin\left(\frac{\sigma_\phi}{2}\right)$$

Since the RMS longitudinal emittance (68% of the beam) is equal to:

$$\varepsilon_{p_{RMS}} = \frac{2\pi\, \Omega\, \sigma_T p}{\omega_{RF}\eta} \sin\left(\frac{\omega_{RF}\sigma_T}{2}\right)$$

the mixing factor can be written as:

$$M(f) = \sqrt{\frac{\pi}{2}}\left(\frac{f_o}{f}\right)\left(\frac{p}{\eta\varepsilon_{p_{RMS}}}\right)\sigma_T$$

<u>RF Duty Factor</u>

Bunching the beam with RF increases the particle density that the cooling system samples. The distribution of particles in the RF bucket is approximately Gaussian:

$$\frac{\delta N}{\delta t} = \frac{1}{h}\frac{N_p}{\sqrt{2\pi}\,\sigma_T}\, e^{-\frac{1}{2}\left(\frac{t}{\sigma_T}\right)^2}$$

where N_p is the total number of particles in the machine. The peak density is:

$$\left(\frac{\delta N}{\delta t}\right)_{peak} = \frac{1}{h}\frac{N_p}{\sqrt{2\pi}\,\sigma_T}$$

The particle density for a DC or coasting beam is:

$$\left(\frac{\delta N}{\delta t}\right)_{DC} = \frac{N_p}{T_{rev}} = N_p f_o$$

The duty factor is defined as the ratio of the DC beam density to the peak beam density:

$$D = \frac{\left(\frac{\delta N}{\delta t}\right)_{DC}}{\left(\frac{\delta N}{\delta t}\right)_{peak}}$$

which can be rewritten as:

$$D = \sqrt{2\pi}\, h\, f_o \sigma_T$$

The Duty factor for the RF harmonic and revolution frequency used in this study is:

$$D(\%) = (0.315)\,\sigma_T(nS)$$

Because the cooling system sees an increase density of particles with a bunched beam,the cooling time becomes:

$$\frac{1}{\tau} = \frac{WD}{N_p}\frac{1}{M+U}\left(\frac{g}{g_{opt}}\right)\left(2 - \frac{g}{g_{opt}}\right)$$

Data Summary

A summary of the bunched beam study data is shown in the following table.The Duty and the Mixing factors were calculated assuming that the bandwidth of the cooling system for a bunched beam is the same as the DC or coasting beam bandwidth (2.46 GHz). Because the Signal/Noise measurements for this study were not trustworthy and considering the fact that the Signal/Noise is a function of the vertical emittance which is changing during the measurement, the U factor was not included in the calculations for the Duty factor. Following the data table is a graph of the duty factor as a function of σ_T. The slope of the graph is 0.362 % per nS change of σ_T which can be compared to the theoretical estimate of 0.315 % per nS change of σ_T derived earlier.

Conclusions

The data presented in this study indicates that the cooling duty factor is linearly proportion to the bunch length. Since the mixing factor is also linearly proportional to the bunch length, the cooling time for a given number of particles, cooling gain, and longitudinal emittance should be independent of bunch length. Also there was no evidence of any coherent signals in the 4-8 GHz stochastic cooling system. The following is list of suggestions for future studies on bunched beam cooling.

1. More accurate data on bunch shape in the time domain coupled with more bunch profiles taken during the betatron emittance measurement.
2. More accurate data of the Signal/Noise coupled with more Signal/Noise measurements during the betatron emittance measurement. Also, the Signal/Noise should be measured at other frequencies throughout the cooling band.

3. Cooling with various beam intensities. As an alternative to No. 2, the studies could be made with large beam intensities so that the signal to noise factor is not important.
4. Bunch beam cooling at h=84.
5. Horizontal bunch beam cooling.
6. Longitudinal bunch beam cooling both at 2-4 GHz and 4-8 GHz.

Bunched Beam Cooling Studies 9-1-89

	Before delta P cooling		
RF voltage (Volts)	68 % Half Width Time Spread of the Bunched Beam (nS)	68% Longitudinal Emittance eV-Sec	10 dB Width of Longitudinal Schottky band at h=126 (Hz)
87	149	0.87	1975
570	113	1.31	2710
1200	102.5	1.58	4180
3000	91	1.98	4840

	After delta P cooling			
RF voltage (Volts)	68 % Half Width Time Spread of the Bunched Beam (nS)	68% Longitudinal Emittance eV-Sec	10 dB Width of Longitudinal Schottky band at h=126 (Hz)	Longitudinal Emittance Cooling Time (min)
87	149	0.87	1445	#DIV/0!
570	72	0.54	1920	11.3
1200	78	0.92	2500	18.7
3000	69	1.15	4025	18.4

	Before delta P cooling		After delta P cooling	
RF voltage (Volts)	Signal Suppression (dB)	g/gopt	Signal Suppression (dB)	g/gopt
87	2.66	0.358	2.97	0.408
570	1.6	0.202	2.19	0.287
1200	1.88	0.242	2.8	0.380
3000	1.48	0.186	2.4	0.318

Bunched Beam Cooling Studies 9-1-89

	Before delta P cooling		After delta P cooling	
RF voltage (Volts)	Shottky line (S+N) at 4 GHz (dBm)	U Using Time domain data	Shottky line (S+N) at 4 GHz (dBm)	U Using Time domain data
87	-78.56	1.99	-78.37	1.88
570	-80.95	2.21	-80.58	2.97
1200	-80.46	1.39	-79.71	1.40
3000	-81.51	1.48	-80.49	1.30
Noise Floor	-85.00			

			Before delta P cooling	After delta P cooling
RF voltage (Volts)	Sychrotron Frequency (Hz)	Beam Current (mA)	Cooling time (Sec)	Cooling time (Sec)
87	5.2	5.65	696	706
570	13.4	4.77	663	1012
1200	19.5	6.61	688	873
3000	30.8	6.82	732	998

	Before delta P cooling		After delta P cooling	
RF voltage (Volts)	Duty Factor Using Time domain (%)	Mixing Factor Using Time domain data	Duty Factor Using Time domain data (%)	Mixing Factor Using Time domain data
87	58	10.37	52	10.37
570	42	5.21	31	8.01
1200	36	3.93	25	5.11
3000	31	2.79	19	3.64

NOTE: Signal to noise measurement is not used in calculation.

Average Duty Factor = 0.362 % per ns of RMS half width bunch length

Cooling Duty Factor vs RMS Bunced Beam Length

Calculated Without Using the Noise to Signal Factor